General Geometric Theory of Attitude Determination from Directional Sensing

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Theme

A DIVERSITY of attitude sensors is used on different spacecraft. Attitude determination softwares have always been developed individually for individual spacecraft, based on its particular dynamic behavior and the particular attitude sensors used. There is a need for the development of a standardized attitude determination and error analysis software package applicable to the many different spacecraft. This paper is addressed to the measurement subsystem of such a program, and to the relation of the attitude sensor measurements to the spacecraft orbit.

Contents

Apart from self-contained inertial instruments which are important during thrusting periods, but are secondary for definitive attitude determination, most attitude sensors are on-board directional sensors which measure external reference directions relative to the spacecraft. Examples of such sensors include magnetometers, interferometers, horizon scanners, star trackers, etc. The reference direction sensed may be an ambient field vector such as the geomagnetic field vector. More frequently, it is a spacecraft to object vector as traced by an electromagnetic radiation path. If the object is a distant star, the vector is a fixed direction in inertial space. If the object is a landmark on Earth, the vector depends on the position or orbit of the spacecraft. In that case the directional measurements are related to both the spacecraft attitude and orbit and would be useful in both attitude and orbit determination.

Although different sensors may be made of different hardwares and may sense different phenomena, their measurements constitute the following fundamental information: and the angle between two lines, one of which is a reference direction and the other a spacecraft-fixed direction; and the angle between two planes, one of which contains the reference direction and a spacecraft-fixed direction, the other is spacecraft-fixed and contains the same spacecraft-fixed direction. This information may serve as standard measurement inputs to an attitude determination and error analysis program. Let $\bar{R}$ be a unit vector in the reference direction and $\bar{K}$ a spacecraft-fixed unit vector. Then the above directional information may be represented by the two vector products; i.e., $\bar{R}\cdot\bar{K}$ and

$$\frac{\bar{R}\times\bar{K}}{|\bar{R}\times\bar{K}|}=\frac{\bar{R}\cdot\bar{J}}{[1-(\bar{R}\cdot\bar{K})^2]^{1/2}}\bar{i}-\frac{\bar{R}\cdot\bar{i}}{[1-(\bar{R}\cdot\bar{K})^2]^{1/2}}\bar{j},$$

where $\bar{i}$, $\bar{j}$ and $\bar{K}$ form a set of spacecraft-fixed right-handed orthogonal unit vectors. These measurements may be related to the spacecraft attitude and the measurement errors by the following equations

$$y_1(t+\Delta t)=[R(t+\Delta t)^I+\Delta R^I]^T \times[A_{I/B}(t+\Delta t)][D^B+\Delta D^B]+\Delta_1 \quad (1)$$

$$y_2(t+\Delta t)=[R(t+\Delta t)^I+\Delta R^I]^T[A_{I/B}(t+\Delta t)][J^B+\Delta J^B] / [1-([R(t+\Delta t)^I]A_{I/B}(t+\Delta t)][K^B+\Delta K^B])^2 \quad (2)$$

$$y_3(t+\Delta t)=[R(t+\Delta t)^I+\Delta R^I]^T[A_{I/B}(t+\Delta t)[I^B+\Delta I^B] / [1-(R(t+\Delta t)^I+\Delta R^I][A_{I/B}(t+\Delta t)][K^B+\Delta K^B])^2]^{1/2}+\Delta_3 \quad (3)$$

where the y's are actual or equivalent sensor outputs; $[I^B]$, $[J^B]$, $[K^B]$ are a set of spacecraft-fixed orthogonal unit vectors; $[D^B]$ is a spacecraft-fixed unit vector representing a sensor sensitivity axis; $[R^I]$ is the reference direction unit vector expressed in terms of a set of inertial axes; $[A_{I/B}]$ is the direction cosine matrix relating the spacecraft axes to the inertial axes and may be expressed in terms of other spacecraft attitude parameters such as Euler's angles and quaternions; t is the measurement time; the Δ's are the errors; $[\ ]^T$ represents the transpose of $[\ ]$; a product such as $[R^I]^T[A_{II/B}][D^B]$ is the matrix representation of the scalar product $\bar{R}\cdot\bar{D}$, with $\bar{R}$ and $\bar{D}$ represented in the inertial and spacecraft axes, respectively. Notice y_2 and y_3 should be constrained by the relation $y_2{}^2+y_3{}^2=1$. Equations (1–3) serve as basic observation equations. Various quantities of interest are clearly delineated. The left-hand sides of these equations, the y's are either direct sensor outputs or are quantities easily computed from the sensor outputs. The right-hand sides are what "the sensors are supposed to sense." The spacecraft-fixed directions generally depend on the instrument alignment and when expressed in spacecraft coordinates, are independent of the spacecraft motion. These measurements are not related to the rate variables. Any information concerning the spacecraft orbit must enter through $[R^I]$ and that concerning the attitude through $[A_{I/B}]$. In the usual attitude determination problem $[R^I]$ is assumed known and $[A_{I/B}]$ is the unknown to be determined. If the attitude $[A_{I/B}]$ is known and $[R^I]$ is an unknown spacecraft to object vector, these equations can be used to determine the orbit. In a combined attitude/orbit determinaiton problem, both $[A_{I/B}]$ and $[R^I]$ are treated as unknowns.

Different instruments have different errors. But as long as attitude estimates are derived from these basic measurement equations, instrument errors must enter, or transform into errors in these equations. In these equations, the errors considered are: instrument reading errors, Δ_1, Δ_2, Δ_3; reference direction error $[\Delta R^I]$; instrument alignment errors $[\Delta D^B]$,

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$[\Delta I^B]$, $[\Delta J^B]$, $[\Delta K^B]$; timing error, Δt. Let us investigate these errors in a little more detail. The instrument reading errors may be biases and/or random fluctuations. since the y's may be converted rather than original measurements, Δ_1, Δ_2, Δ_3 may be equivalent biases, etc. But they are readily calculated once the corresponding errors in the original measurements are given. Most of the time the reference direction $\bar{R}$ is the line of sight from the spacecraft to a "spacemark," and may be written as $\bar{R}=(\bar{\rho}-\bar{E})/|\bar{\rho}-\bar{T}|$, where $\bar{\rho}$ is the orbital position vector and $\pm\bar{T}$ is the position vector of the spacemark independent of the orbital position. Therefore, to first order one may write the reference direction error as

$$[\Delta R^I] \approx \frac{I}{[\rho^I]-[T^I]}\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - [R^I][R^I]^T\right)$$

$$\times([\Delta\rho^I]-[\Delta T^I])$$

which exhibits explicitly the effect of orbital error and spacemark position error on the error in the reference direction.

If the sensor has a single sensitivity axis, the alignment error of this axis depends on two parameters. On the other hand, for a carefully aligned instrument package with several sensitivity axes, it may often be assumed that alignment errors are instrument package mounting errors. The small mounting errors may represented by a misalignment vector

$$[\epsilon^B]^T=[\epsilon_1\epsilon_2\epsilon_3] \text{ such that } [\Delta(\,)^B]=[(\,)^B]\times[\epsilon^B]$$

where $(\,)=D,I,J,K$, etc., as the case may be, and $-\epsilon_1$, $-\epsilon_2$ and $-\epsilon_3$ represent small misaligning rotations of the instrument package about the three spacecraft body axes. In this case there will only be a maximum of three misalignment error parameters regardless of how many instruments there are in the instrument package.

Timing error is frequently one of the most important sources of error, and it enters into both the reference direction and the direction cosine matrix. Since the attitude motion generally has a much shorter time constant than that of the orbital motion, one may disregard the error in reference direction due to timing error. If the timing error is small

$$[A_{I/B}(t+\Delta t)] \cong [A_{I/B}(t)] + [A_{I/B}(t)][\tilde{\Omega}^B]\Delta t$$

where $[\Omega^B]$ is the spacecraft angular velocity tensor. An important part of any attitude/orbit determinaiton and error analysis program consists of the measurement partial derivative. It is a tedious but straightforward task to obtain them from the measurement and error equations given above, and they are listed in the full paper.

The basic observation equations have been given. If measurement errors are not considered, any meaningful observation corresponds to some attitude. A remaining question is, "What combinations of these basic measurements are required to resolve the attitude unambiguously?" From the kinetics of a rigid body one knows that in order to determine the spacecraft attitude: a complete knowledge of two spacecraft-fixed or spacefixed directions are sufficient; and at least three independent measurements related to two reference directions are required.

However the resolution of spacecraft attitude from partial measurements of two or more reference directions, and from inaccurate measurements, becomes much more difficult. A graphical construction, with directions and attitudes represented as points on a unit sphere, provides a great deal of insight to the problem, and such constructions are illustrated in the full paper. The two basic measurements discussed before represent "small circle" and "half great circle" respectively on the unit sphere, and intersections of these circles define directions. It is relatively easy to see graphically the situations under which the intersections are nonunique or ambiguous. This graphical construction has been applied to explain the difficulty of accurately determining the attitude of IMP – J during an important part of its mission, even though a rather sophisticated statistical attitude estimation scheme was used.